The usage of technology to revive classical topics in mathematics

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The multi-purpose nature of a Computer Algebra Systems (CAS) enables to build bridges between mathematical domains which are generally taught separately. The various registers of representation provided by CAS and other kinds of software, and their versatility enabling to switch between them correspond to the required skills for building these bridges. For this, technology has to be used not as a bypass for a lack of theoretical knowledge, but as a facilitator to enhance new mathematical knowledge and more profound conceptual understanding.

A technology rich environment provides a working framework for experimentation, conjecturing and automated proofs (mostly using algorithms based on computations of Gröbner bases).

In this lecture, I will illustrate this in two different situations:

1. The computation of parametric definite integrals is an important topic in applied science. The usage of a CAS, in particular of the scaffolding tools which are sometimes built-in, may lead to the discovery of different ways of computing the same integral. A by-product may be the derivation of integral presentations of combinatorial objects, such as Catalan numbers, and the derivation of combinatorial identities.
2. A characteristic of differential geometry is the mixing of algebraic, geometric and analytic skills. The examples will show situations of interest to mathematicians, applied scientists and engineers: the study of envelopes of 1-parameter families of plane curves, using a CAS as system for Dynamic Geometry, and the study of isoptic curves of plane curves. In this last case, technology enabled to find a transition from conic sections to toric sections which did not exist in the literature.

The lecture fits the curriculum for undergraduates, sometimes for advanced high-school students. The second one provides an extension of the curriculum for undergraduates.